

# Statistical hadronisation phenomenology

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The analyses of hadron production in the framework of the statistical hadronisation model are reviewed. The analysis of average multiplicities in collisions at relatively low centre-of-mass energy confirms previous findings, namely the universality of hadronisation temperature and of strange to non-strange quark production rate. The study of transverse momentum spectra of identified hadrons allows a further determination of the hadronisation temperature which is found to be compatible with that obtained from fits to average multiplicities.

## 1. INTRODUCTION

The idea of a statistical approach to hadron production in high energy collisions dates back to '50s [1] and '60s [2] and it has been recently revived by the observation that hadron multiplicities in  $e^+e^-$  and  $pp$  collisions agree very well with a thermodynamical-like ansatz [3–5]. This finding has also been confirmed in hadronic collisions and it has been interpreted in terms of a pure statistical filling of multi-hadronic phase space of assumed pre-hadronic clusters (or fireballs) formed in high-energy collisions, at a critical value of energy density [5–7]. In this framework, temperature and other thermodynamical quantities have a purely statistical meaning and do not involve the existence of a hadronic thermalisation process through multiple collisions on an event-by-event basis.

So far, this proposed statistical cluster hadronisation model has been mainly tested against measured abundances of different hadron species for a twofold reason. Firstly, unlike momentum spectra, they are quantities which are not affected by hard (perturbative) QCD dynamical effects but are only determined by the hadronisation process; indeed, in the framework of a multi-cluster model, they are Lorentz invariant quantities which are independent of individual cluster's momentum. Secondly, they are fairly easy to calculate and provide a very sensitive test of the model yielding an accurate determination of the temperature. It is now a quite natural step to test further observables and to assess their consistency with the results obtained for multiplicities. One of the best suited observables in this regard is the transverse momentum of identified hadrons (where transverse is meant to be with respect to beam line in high energy hadronic collision, and thrust or event axis in high energy  $e^+e^-$  collisions) because, amongst all projections of particle momentum, this is the most sensitive to hadronisation or, conversely, the least sensitive to perturbative QCD dynamics.

In this paper, after a brief sketch of the statistical hadronisation model, the results

obtained in a comprehensive analysis of average multiplicities and transverse momentum spectra of identified hadrons in various elementary collisions are summarized. A more detailed description of both the model and the analysis can be found in ref. [8].

## 2. HADRON MULTIPLICITIES

In order to obtain quantitative predictions in the model, the assumption of local statistical equilibrium is not enough because average values of physical observables also depend on clusters configurations in terms of volume, four-momentum and charges. Thus, the probabilities of clusters configurations are needed, but they are governed by the particular dynamical evolution of the process which is beyond the statistical ansatz. For Lorentz-invariant observables such as average multiplicities, one can take advantage of their independence of cluster momenta and try to establish a mathematical equivalence between the actual system of clusters and one *equivalent global cluster* (EGC) with volume and quantum numbers equal to the sum of single volumes and quantum numbers. This equivalence would lead to a dramatic reduction of the number of model's free parameters and one would be essentially left with volume and mass of the EGC. However, for this equivalence to apply, a special form of cluster masses and charges distribution for fixed volumes is required [8]. If this is the case, the average multiplicity of the  $j^{\text{th}}$  hadron reads:

$$\langle n_j \rangle = \int dM \int dV \chi(M, V) n_j(M, \mathbf{Q}, V) \quad (1)$$

where  $n_j(M, \mathbf{Q}, V)$  is the average multiplicity for fixed EGC's mass, quantum numbers  $\mathbf{Q} = (Q_1, \dots, Q_n)$  and volume and  $\chi$  is the EGC's mass and volume distribution. If the EGC is sufficiently large, the canonical ensemble can be introduced through a saddle-point approximation and the above multiplicity can be written as:

$$\langle n_j \rangle = \int dT \int dV \zeta(T, V) n_j(T, \mathbf{Q}, V) \simeq n_j(\bar{T}, \mathbf{Q}, \bar{V}) \quad (2)$$

where  $\bar{T}$  and  $\bar{V}$  are meant to be mean values independent of hadron species. The last equality is a good approximation if  $\zeta$  is a peaked function of  $T$  and  $V$ . It must be stressed that, in this procedure, temperature can be a well defined concept only in a global sense, i.e. for the EGC, while individual clusters might be so small that a microcanonical description is essential. Otherwise stated,  $T$  could be introduced at a global level even though not locally defined.

In order to reproduce the experimentally measured multiplicities, an extra strangeness suppression parameter must be introduced. This has been usually done [3–5] by means of a factor  $\gamma_s^{n_s}$  (where  $n_s$  is the number of strange quarks in the hadron) multiplying, in the Boltzmann limit, the full-equilibrium primary multiplicity on the right-hand-side of eq. (2). For some of the considered colliding systems, a new parametrisation has been used in which the number of newly produced  $s + \bar{s}$  constituent quarks is considered as an additional charge to be fixed into the final hadrons. The number of produced  $s\bar{s}$  pairs are allowed to fluctuate poissonianly around a mean value  $\langle s\bar{s} \rangle$ , taken as a further free parameter, so that the primary average multiplicity actually reads:

$$\langle n_j \rangle^{\text{primary}} = \frac{VT(2J_j + 1)}{2\pi^2} \sum_{K=0}^{\infty} \frac{e^{-\langle s\bar{s} \rangle} \langle s\bar{s} \rangle^K}{K!} \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{m^2}{n} K_2\left(\frac{nm}{T}\right) \frac{Z(\mathbf{Q} - \mathbf{q}_j)}{Z(\mathbf{Q})} \quad (3)$$

where  $\mathbf{Q} = (Q, N, S, N_S)$  and  $\mathbf{q}_j = (Q_j, N_j, S_j, N_{Sj})$  are the quantum numbers vectors with components electric charge, baryon number, strangeness and number of strange quarks associated with the initial state and with the  $j^{\text{th}}$  hadron respectively and the  $Z$  are the canonical partition functions. The free parameters in Eq. (3) are the temperature  $T$ , the volume  $V$  and the mean number of newly produced  $\langle s\bar{s} \rangle$ . From these parameters, one can calculate the Wroblewski parameter  $\lambda_S = 2\langle s\bar{s} \rangle / (\langle u\bar{u} \rangle + \langle d\bar{d} \rangle)$ , the ratio between newly produced  $s\bar{s}$  pairs and light quark pairs by using the fitted primary multiplicities of all the hadron species.

The results of the fit to average multiplicities are summarized in figs. 1 and 2 showing the best-fit values of  $T$  and  $\lambda_S$  for several kinds of collisions. The striking feature of both plots is the fair constancy of the quantity over two orders of magnitude of centre-of-mass energy regardless of the initial colliding particles. Indeed,  $T$  seems to rise as centre-of-mass energy decreases, but the genuineness of this effect is not established yet.

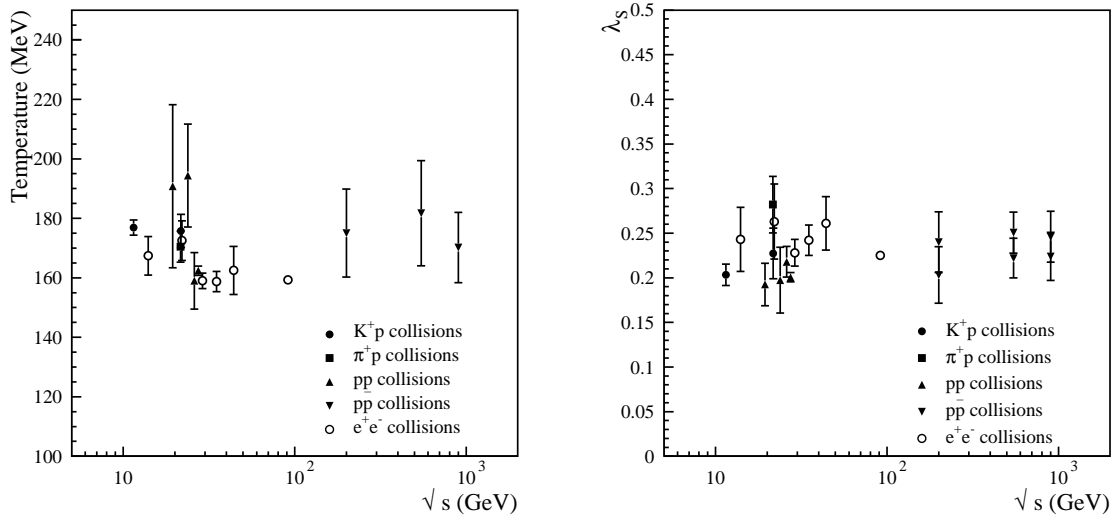


Figure 1. Temperatures fitted with hadron multiplicities [5,8] in high energy collisions. The  $e^+e^-$  point at  $\sqrt{s} = 91.2$  GeV has been calculated for this conference.

Figure 2. Wroblewski factor  $\lambda_S$  calculated with fitted primary hadron multiplicities [5, 8]. The  $e^+e^-$  point at  $\sqrt{s} = 91.2$  GeV has been calculated for this conference.

### 3. STUDY OF TRANSVERSE MOMENTUM SPECTRA

The main goal of the analysis of transverse momentum spectra of identified hadrons is the assessment of their consistency with the prediction of the statistical hadronisation model. This has a twofold implication: firstly, spectrum shapes of different hadrons at a given centre-of-mass energy should be described by essentially the same parameters; secondly, the best-fit  $T$ 's should be in agreement with the temperature fitted with hadron

multiplicities. In view of this objective, a natural requirement for the data set for a given collision and centre-of-mass energy, is the existence of a considerably large sample of both measured transverse momentum spectra and integrated multiplicities of different hadron species. Furthermore, centre-of-mass energy must be high enough to allow the use of a canonical formalism, what is expected to occur above roughly  $\sqrt{s} \approx 10$  GeV [8]. Essentially, four collision systems fulfilling these requirements have been found:  $K^+p$  at  $\sqrt{s} = 11.5$  and  $\sqrt{s} = 21.7$  GeV,  $\pi^+p$  at  $\sqrt{s} = 21.7$  GeV and  $pp$  at  $\sqrt{s} = 27.4$  GeV.

Provided that the EGC exists and that the average transverse (with respect to beam line in hadronic collisions and event axis in  $e^+e^-$  collisions) four-velocity  $\bar{u}_T$  of clusters is sufficiently small, the transverse momentum spectrum of the  $j^{\text{th}}$  hadron species can be written, in the canonical ensemble, as:

$$\begin{aligned} \left\langle \frac{dn_j}{dp_T} \right\rangle &= \frac{\bar{V}(2J_j + 1)}{2\pi^2 \sqrt{1 + \bar{u}_T^2}} \sum_{n=1}^{\infty} (\mp 1)^{n+1} m_T p_T K_1 \left( \frac{n \sqrt{1 + \bar{u}_T^2} m_T}{T} \right) I_0 \left( \frac{n \bar{u}_T p_T}{T} \right) \\ &\times \frac{Z(T, \mathbf{Q} - n\mathbf{q}_j, \bar{V})}{Z(T, \mathbf{Q}, \bar{V})} + \sum_k \left\langle \frac{dn_j}{dp_T} \right\rangle^{k \rightarrow j} \end{aligned} \quad (4)$$

The first term on the right hand side is the primary spectrum while the second term is the contribution to the spectrum due to the decays of hadron  $k$  into the hadron  $j$  either directly or through intermediate steps. The latter has been calculated by means of a newly proposed mixed analytical-numerical method [8].

The analysis is described in full detail in ref. [8]; hereby, the main results are briefly summarized. The free parameters in the formula (4) are essentially  $T$  and  $\bar{u}_T$ , the others being fixed by the multiplicity fit. For each identified hadron, there are several  $\chi^2$  local minima located along a band in the  $T - \bar{u}_T$  plane. Thus, in order to single out a best-fit solution, the minimum nearest to the *chemical* temperature (i.e. fitted with multiplicities)  $1\sigma$  band is chosen. The use of this method does not allow to give a conclusive answer to the question whether there is independent consistency between the temperatures governing the  $p_T$ -slopes and multiplicities respectively. Still, it is possible to test the universality of  $T$  and  $\bar{u}_T$  for different hadron species at a given centre-of-mass energy. This is indeed shown in fig. 4 demonstrating that the fitted parameters for eight different particles in  $pp$  collisions are found to be in good agreement with each other. On the other hand, some difficulties have been found in a similar analysis of hadrons measured in  $K^+p$  and  $\pi^+$  collisions at  $\sqrt{s} = 21.7$  GeV.

## 4. CONCLUSIONS

A comprehensive analysis of multiplicities and transverse momentum spectra of identified hadrons in several high energy collisions confirms the agreement of the data with the predictions of statistical hadronisation model. Particularly, the temperature estimated with these two observables are fully compatible with each other, which is an indication in favour of one of the key predictions of the model. The extracted temperature is fairly constant throughout with a value of about  $\simeq 160$  MeV in the high energy limit, which is amazingly close to that determined in similar analyses in heavy ion collisions [9]. The

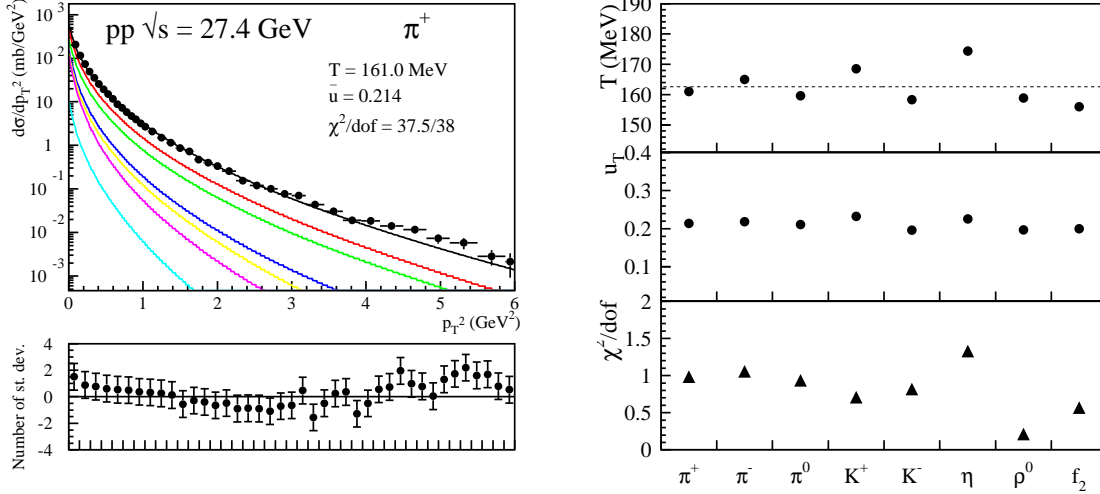


Figure 3. Fitted  $\pi^+$  transverse momentum spectrum in pp collisions at  $\sqrt{s} = 27.4$  GeV. The grey-scaled solid lines are the cumulative contributions of the decays of various sets of parent hadrons. Below: residuals distribution.

Figure 4. Fitted temperatures and cluster spectrum in pp collisions at  $\sqrt{s} = 27.4$  GeV with identified hadrons spectra. Also shown the minimum  $\chi^2$  values.

universality of  $T_{\text{had}}$  and its very value, close to the expected QCD critical temperature for deconfinement and chiral symmetry restoration, strongly suggest that hadronisation itself is a critical phenomenon occurring at a peculiar value of (local) energy density [5–7]. The production ratio of s quark with respect to light quark is fairly constant as well, with a value around  $\approx 0.23$ .

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